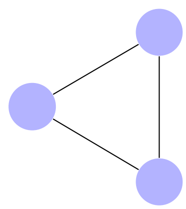
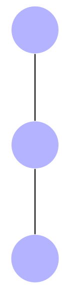
Exercise 1: (representations and graph isomorphisms)
Given the following 2 graphs:





For each of the graphs, in the following called G:

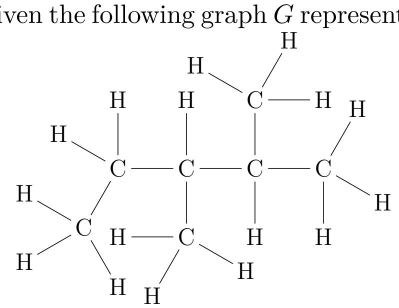
1. How many different representations (adjacency matrices) can you find for graph G? (Let r_G be this number)

2. Chose an adjacency matrix B. How many permutation matrices can you find, such that $B = P(PB)^T$? (Let p_G be this number).

   Note: p_G corresponds to the number of isomorphisms from B to itself, which is also called an automorphism. You could chose any pair of representations A and B, and finding the number of different P for which $A = P(PB)^T$ holds, the result will always be p_G.

3. What is the product of the number of representations r_G and the number of isomorphisms p_G?

Exercise 2: (Wiener index and boiling points)
Given the following graph G representing the chemical compound 2,3-dimethylpentan:



1. Determine the edge-weight matrix of the graph of the carbon backbone.

2. Determine the distance matrix.

3. Determine the Wiener-Index.

4. Determine the number of shortest paths of length 3.

5. Determine the value p_0 and w_0 of the formula for predicting the boiling point for this compound.

6. Determine the estimated boiling points and compare it to the real boiling point.

7. What is the asymptotic worst case performance for finding the distance matrix based on repeated squaring?

8. Do you know a method that has a better asymptotic worst case performance?

Exercise 3: (From random polygon to an ellipse)

Given the matrices

$$M_3 = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

and

$$M_4 = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

1. Which of both matrices is invertible?

2. Compute the determinant of M_3 and M_4.

3. Are the columns of M_3 independent? Are the columns of M_4 independent?

4. Draw an equilateral triangle with points (x_1^k, y_1^k), (x_2^k, y_2^k), and (x_3^k, y_3^k). Assume the triangle is a result of $M_3 \cdot x^{k-1}$ and $M_3 \cdot y^{k-1}$ as presented in the lecture. Ignoring normalization, find x^{k-1} and y^{k-1}. Can you find several solutions for x^{k-1} and y^{k-1}?

5. Draw a square with points (x_1^k, y_1^k), (x_2^k, y_2^k), (x_3^k, y_3^k), and (x_4^k, y_4^k) . Assume the square is a result of $M_4 \cdot x^{k-1}$ and $M_4 \cdot y^{k-1}$ as presented in the lecture. Ignoring normalization, find x^{k-1} and y^{k-1}. Can you find several solutions for x^{k-1} and y^{k-1}? What is the conclusion wrt. the (non-)existence of an inverse of M_4?

Exercise 4: (From random polygons to an ellipse)
Given vector $v = (0, 3, -1, 11, -3)$.

1. Determine $w = v - \overline{v}$, where $\overline{v}$ is a vector where each entry is the mean of all values v_i.

2. Determine $\frac{w}{||w||_2}$, where $|| \cdot ||_2$ refers to the 2-norm.

3. What is the length of vector $\frac{w}{||w||_2}$?