DM561 – Linear Algebra with Applications

- The reexam consists of two separated assignments:
  - *Theory assignment*:
    - Specifications in this document
    - **Deadline: Monday, March 25, 2019 at noon**
    - Hand in PDF via BlackBoard SDU Assignments.
  - *Applications assignment*:
    - Specifications at the course web page https://dm561.github.io/: assignments: asg1, asg2, asg3, asg4, asg5, asg6
    - **Deadline: Monday, April 1, 2019 at noon**
    - Hand in via git push as during the course.
- You are expected to work individually on the assignments.
- Deadline extensions will not be conceded.

Theory Assignment

You may use the course book, the slides from the lecture, and notes from the lectures and the exercises. You are allowed to use tools such as Python or Maple to assist you in calculations, but you must write the calculations out in full so that they can be followed by the examiners. It is not sufficient to present an answer, you must show how you found it. If you use a theorem from the book (or the slides), make a reference to it.

For questions, you may contact the lecturer by e-mail marco@imada.sdu.dk.
You may answer in Danish or English. Your solution must be handed in as a single PDF file clearly stating your **full name and birth date** on the front page. You have to hand in using SDU Assignment before Monday, March 25 at noon.

Problem 1

Consider the following system of linear equations in variables $x, y, z, w \in \mathbb{R}$.

$$\begin{aligned} x + 2y &= 0 \\ -2w + x &= 4 \\ 2z + 2y &= -1 \\ 4x + 8y &= 0 \end{aligned}$$

1. Write the augmented matrix of this system.
2. Reduce this matrix to row echelon form by performing a sequence of elementary row operations.
3. Solve the system and write its general solution in parametric form.

Problem 2

Consider the following matrix

$$M = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 2 & 2 & 2 \end{bmatrix}.$$

1. Find M^{-1} by performing row operations on the double matrix $[M \mid I]$.
2. Is it possible to express M as a product of elementary matrices? Explain why or why not.

Problem 3

Consider the following matrix

$$M = \begin{bmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \\ x_7 & x_8 & x_9 \end{bmatrix}$$

with $x_i \in \mathbb{R}$ for all i and $\det(M) = 10$. Let N be given by

$$N = \begin{bmatrix} x_4 & x_5 & x_6 \\ x_1 & x_2 & x_3 \\ x_7 + 2x_1 & x_8 + 2x_2 & x_9 + 2x_3 \end{bmatrix}$$

1. Find $\det(N)$.
2. Find $\det\big(N^2\big)$.

Problem 4

Let P_1 be the vector space consisting of polynomials of degree at most one with real coefficients with the usual addition and scalar multiplication and with the inner product $\langle p, q\rangle = p(0)q(0) + p(1)q(1)$.

1. Let $T : P_1 \to P_1$ be a linear transformation with $\text{rank}(T) = 1$. Is S an isomorphism?
2. Consider the two polynomials $f, g \in P_1$ with $f(x) = x + 1$ and $g(x) = 2x + \frac{1}{2}$. Do these form a basis for P_1? If so, is it an orthonormal basis?
3. Is it possible to find two orthogonal polynomials $f, g \in P_1$ and a scalar $t \in \mathbb{R}$ such that $t \cdot f = g$? Find such an f, g and t or show that it is not possible.

Problem 5

Consider the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by

$$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ y - 3x \end{pmatrix}$$

1. Find a basis for the kernel of T.
2. Find the rank and nullity of T.
3. Find the matrix of T with respect to the basis $B = \left\{ \begin{pmatrix} 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \end{pmatrix} \right\}$. That is, find $[T]_B$.

Problem 6

Consider the following matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 3 & 2 \end{bmatrix}$$

1. Find the eigenvalues for A.
2. For each eigenvalue, find a basis for the corresponding eigenspace.
3. Can you find matrices P and D such that such that $P^*AP = D$ with D diagonal? Find D and P or argue that it is not possible.

——— *End of Examination* ———